CONSTRUCTIVISM: EPISTOMOLOGY TO PRACTICE

JUDITH A MOUSLEY
Faculty of Education, Deakin University, Geelong

CONSTRUCTIVISM AND SCHOOLING

Theory building in classrooms

> The mind not only "holds" beliefs, it perceives, construes and interprets information about the world, then hypothesizes, conjectures and reasons about this information. This results, at times, in decisions, beliefs, knowledge and at other times in confusion, wonder and understandings ... the mind is an intermediary that interprets and directs all perceptions and action ...
>
> (Welman, 1992, p.88)

This paper explores the notion of conceptual change in mathematics classes being a product of theory construction (Carey,1986; Chandler and Boyse, 1982; Gopnik,1984; Welman, 1992) by both teachers and students. Its main thesis is that while there might be a growing recognition of constructivism as epistomology, the institutionalisation of practices which reflect such philosophies is inhibited by our 'knowledge' of what mathematics schooling is.

The way that teachers in classrooms act out their roles, and hence the way they constrain the potential realities of others, is based on theories they have developed about what it is to be a teacher or a learner, about what it is to do mathematics and even about the nature of mathematics itself. These theories have been socially constructed: they have been built up from prior experience, itself tempered by active interpretation and linguistic interaction. If we are to accept the principle that knowledge is not passively received, but actively interpreted by the cognising subject (Lerman, 1989; Wheatley, 1991), we need to research interpretation of personal rôles in classrooms. If we acknowledge that subjects construct viable explanations of experiences (von Glasersfeld, 1987), we should study how teachers make sense of and theorise about their own impacts on the meso-world of the classroom.

Developing understandings of what it is to educate

The assumption of transmission-based pedagogies that major conceptual changes in students can be brought about through expressive action by teachers (paired with relatively inactive reception by learners) has been brought into question by the constructivist movement[1]. This cluster of perspectives is based around the alternative notion that learners actively establish learning goals, pursue these by selectively interacting with

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[1] There is good reason for the irony that Ackerman (in press) notes - that for mathematics education the meanings of the labels "constructivism" and "constructivist" can never been entirely clear. If we believe that knowledge is constructed by individuals, we cannot make a claim that there is one form of constructivism, for this would be self-contradictory.

communications from others, then create for themselves appropriate schemas of understanding[2]. Meaning will be tempered by current understandings, individual ways of organising knowledge and personal experience of both the concepts and social situation being interpreted. Thus received ideas are sifted from the broader communication media, fitted with prior experiential understandings, reconstrued and used as a basis for further action (including subsequent building of new ideas) by individuals.

Central to this view of the development of personal constructs (Kelly, 1955) is the notion of knowledge being a product of choice (Gergen, 1985), a concept closely related to the notion of ownership of what is learned (Labinowicz, 1985; Kamii, 1985; Steffe, 1987, 1990). Choices, however, are not consciously made and not free of constraints: the shifting set of theories on which constructivism is based share the notion that constructive action within the mind of an individual is a social artifact (see, for instance; Gergen, 1985; Sullivan,1984; Watzlawick, 1984; Wexler,1983). Learning, for the constructivist, is representational theory-building involving re-organisation and re-building of communications to fit with developing schema (von Glasersfeld, 1989).

Creating space for construction of concepts

As Pateman and Johnson (1990) assert, constructivist ideas have influenced pedagogical practice. They note that the movement is impacting on the planning, enaction and evaluation of learning activities. If we believe that students need to construct mathematical concepts, then we need to create the temporal, discursive, and cognitive space for this to happen[3].

This paper aims to link epistomology with practice by identifying some moments where teachers or students create space for themselves and others. The use of the word 'moments' should not imply that these times are discrete or independent: the complexities of classroom interaction defy both iconic and linguistic simplicity. 'Moments' was chosen to reflect the immediate nature of split-second periods in which the potential of any lesson is re-shaped.
The following table lists those moments identified in the recent research project outlined below. It is not suggested that the moments are either linear or paired: they form a dynamic web of interaction. It should be noted that both teachers and students are practitioners in that both contribute to pedagogical practices and outcomes.

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[2] Even the radical constructivist (see, for instance Kamii, 1985, Sutching, 1992; von Glasersfeld, 1990) does not deny the role of teachers in offering knowledge verbally. Constructivism merely shifts the focus to examination of the ways that discursive interactions are handled through perceptive and active cognition.

[3] This is not as easily done as imagined: many writers about constructivist theories (see for instance Ackerman, in press; Duckworth, 1987) note the practical difficulties of putting such theories into action given the culture of classrooms.

Teachers		Students
Interpreting		
Planning		
Presenting	--------------------------->	Interpreting
Supervising		Planning
Evaluating		Performing
		Presenting
		Evaluating

In previous research (Mousley 1990a, 1990b, 1991b), I have explored constructivist classrooms where teachers allow mathematical activity to be planned, organised and controlled largely by children; but these classrooms are rare. And even in these classrooms, mathematical theories do not lie in the tasks, no matter who chooses them. Tasks are merely tools, with pedagogical theories-in-use shaping the ways in which they may be used to create different learning opportunities. At each of these moments, different teaching practices support theory construction to varying degrees.

The research project

In exploring the notion of constructivism in mathematics education, Nerida Ellerton, Ken Clements and I were interested to find out how a number of teachers and then groups of their students would interpret and develop a given activity. It seemed that recording the implementation of one activity in a number of mathematics classrooms would throw light on the way that teachers and students had theorised their rôles in the pedagogical process.

Eleven Australian classroom teachers (of a range of grades from Years 1-9) were involved in the project, as well as some teachers from South-east Asian countries. This paper, however, draws on data from only three Australian classrooms from the Year 5-6 range. The teachers demonstrated a variety of teaching styles and pedagogical philosophies: it would be wrong to imply that the observed teachers are constructivists. We were not so interested in teachers' espoused theories as in the ways that ideology is played out in pedagogy.

The project aimed to capture the moments listed above in order to provide data about how classroom actors construed and reconstrued the task. The teachers were given the activity a week before lessons were to be taught, and their reactions were recorded. Immediately before each lesson, teachers were interviewed about their planning strategies and intentions. Field notes were used to record discussions at these stages. Lessons were then observed, with teachers being videotaped as they introduced and concluded the lessons, and some groups of children video- and audio-taped (later transcribed) during the activity. Other groups were observed while working with field notes being taken. A few weeks after the lessons, teachers and students were interviewed and video snippets were used to assist recall of selected incidents. Discussion focused around participants' perceptions of their rôles at different stages of the lesson.

The text outlining the activity for teachers was a mere two sentences, written on an index card:

> From a given piece of cardboard, make a regular shape which holds one cup of birdseed. Make a similar shape which is twice as big.

One would assume that this activity would result in similar lessons in a number of classrooms. However, as von Glasersfeld (1983) points out, communication of concepts is not as straight-forward as we usually assume.

> The underlying process of linguistic communication, ... the process on which ... teaching relies, is usually simply taken for granted. There has been a naive confidence in language and its efficacy. (p. 43)

In fact, there were quite different interpretations by the teachers then various re-interpretations by their students.

Two moments of re-construction

This paper focuses the link between teachers' initial presentations of the activity and the students' re-interpretations of it. It was found that the former interaction impacted markedly on options pupils had for the latter.

Some teachers thought the given task too vague and realised that it might not lead to the discovery of the 'mathematical knowledge' they felt was bound within the task. One teacher (Teacher H, Year 6), for instance, shaped the lesson by writing on the chalkboard "Make a box which holds one cup of seed. Make another box which holds twice as much." At a later date, the researcher (R) interviewed this teacher (TH):

431	R	I am interested ... to know why you thought the shape should be a box. Had you thought about the possibility of making other shapes?
432	TH	Yes, but I wanted to build on this lesson to give them an understanding of volume.
433	R	Good. So they will do that with box shapes?
434	TH	Yes. They have to learn length by width by height and ... well, they couldn't do that with other shapes. ... Oh, I guess they could, but, like cones and other shapes -- I didn't want shapes where they couldn't measure length and width and height.

In planning the activity around a particular learning objective - a formal rule - Teacher H presumably expected all students to take a relatively directed path of "discovery". His pupils later demonstrated an acceptance of this rôle context, clearly displaying characteristics of students waiting to be led.

463	Rob	But it has to be a box. Not a box. He said a cube. That's the same all around. The same size - this way, this way, this way. Ask him how big. Darren, ask him how big to make it.
464	Darren	How big would fit. You've got the cardboard. How big could we make it? It has to hold a cup.
465	Rob	Just ask him, Daz. He knows.
466	Darren	Okay. He knows. (Inaudible) Mr H...

Similarly, Teacher N (Year 5) thought that the activity could be used for children to discover what happens when all three dimensions of a cube are doubled. She first told the children to "Make a cube 5cm by 5cm by 5cm". When they had all finished, she asked them to "make one measuring 10cm by 10cm by 10cm" and to use the birdseed to compare the volumes of the shapes constructed.

Her students also displayed a reliance on the teacher to solve a mathematical problem they had:

134 Rhana It's too big. We were only meant to double it. You went wrong, Karen

135 Karen No. They are right. 5cm this one, 10cm that one. You try it. With the ruler.

136 Rhana Mmm, but there's too much seed. It should be two lots. Why is there seven and a half? It's mad. Two lots would be right. Don't write it down. I'm going to ask her if we did it right.

At this stage, following Rhana's question, the teacher called the class to attention. After a short discussion which led to the teacher expressing the generalisation that "the big one holds eight times as much", she proceeded to give a six-minute explanation of why this is so. No diagrams or concrete materials were employed, but terms such as *third dimension*, *multiply*, *multiples* and *comparative volume* were used. When asked later if she thought that all of the children would have understood, she claimed

231 TN Yes. That is why it is important to have the hands-on work first. Yes. They had seen it with their own eyes. That is why children need to do real things in mathematics - so they understand why the mathematics works.

In introducing and supervising the activity, both of these teachers created limitations to the way the task could be explored. In doing this, they also limited potential rôles students could play in the "discovery" process.

Other teachers, however, recognised that the task could be interpreted by students in different ways and saw this as opening up opportunities for people to take some ownership over the task. When Teacher M (Year 5/6) first saw the activity, for instance, she said

730 TM It will be interesting to see what they make of it. I'm not even going to explain what *regular* means. I wonder if they will ignore the word. ... Seeing the different shapes will be fun. And their reactions when they double different aspects.

Teacher M presented the task as given, by writing the instructions on the board. The groups in her class made various shapes, and some surprisingly difficult explorations were the result.

For instance, a group of three girls had built a square pyramid, left open at the apex so seed could be poured in using a small funnel. They had found that their shape held more than one cup of seed, so traced it onto another piece of card and trimmed the triangular parts of the net gradually until a pyramid of the right size was formed.

748 Silvia — Don't take too much off. Remember ... (inaudible) ... you are not just taking that bit. That long bit. You are taking it four times. No, eight times. There and there and (etc.)

749 Binny — And it's not just thinner. The shape. Look. It gets shorter so you are losing this bit. The top bit every time. The whole pyramid loses the top bit.

(Some trial and error followed)

783 Silvia — Good. That's it. One cup. I'll tape it up. ... Now. Two cups. Now for two cups.

784 Rachael — Two cups. Yes. Or twice as big? Two cups isn't twice as big.

785 Binny — (Inaudible, then laughed.)

786 Silvia — Yes it is. But I know. But ... I know what you think. Like ... like two times the edges. Make the bottom twice as big. The sides too?

787 Binny — Not the sides.

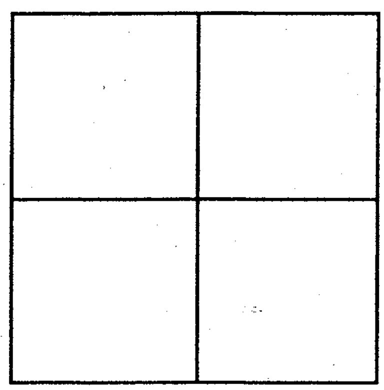
788 Silvia — Why?

789 Binny — We make the bottom twice as big. Right? Longer and wider. But that will be like this and this and this and this. Like four times as big. We need to decide if we want it really twice as big ... or each side double - fourth as big. Anyway, then we could cut down the sides like we did ... so it holds two cups.

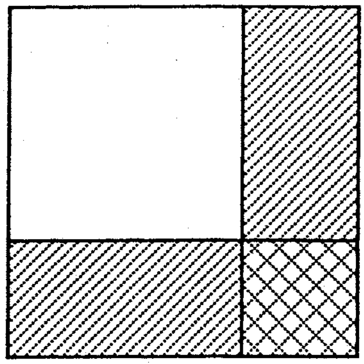
790 Silvia — Mmm. I get it. That way it's twice as big. Well, on the edges anyway.

791 Binny — What edges. What are you on about?

792 Silvia — These edges. The sides. The sides of the square. See. Look. You've got a square, right? (Silvia drew a square.) Then you double this one ... and that one; and you don't have two. What do you have?



793	Binny	Four. I know that.
794	Silvia	Good. Four. So do we want twice or four times? Hey? I reckon only twice. Twice as big.
795	Binny	Yeah, and ...
796	Silvia	And it holds two cups. So its twice as big like that.
797	Rachael	But it needs to be twice as high.
798	Silvia	Look, it doesn't say twice as high on the board. Or two cups. Look.
799	Rachael	No.
800	Silvia	We need to decide ... to make up our minds. What are we going to call twice as big. Jees, I wish we'd started with a box.
801	Binny	No, a box is no good. They are doing a box. Anyway, it's the same. We would be the same. Look, if you do this to the box ... (Binny drew a sketch of a box then one twice as long) ... you've got ...
802	Rachael	Two cups. But it's not twice as big.
803	Silvia	Yes. Yes it is.
804	Rachael	But it's not twice as wide ... or high ... just long. Forget the box. I think we should not double the sides. Of the bottom. We should work out what would make twice as big really. Like. The square. The area of the square.
805	Silvia	Yeah. Mmm.
806	Binny	Times it ... Times it by one and a half. Two is too big because you get four. No. One and a half ...
807	Silvia	That's too big. One point two five - or less? No, probably more that one and a quarter. It's not one and a half - it's less. Look. If you've got this square. Right? and it's half as long again ... and half as long on to this side. (*Hatched areas, below*) Right? That's one and a half. But then you've got this piece *(double-hatched, below)*. That means one and a half is too much.



808	Rachael	Can we work backwards? Jees. No. We don't know how many little squares. Get some graph paper so we can do it on little squares. Like, we count the squares. Then double it.
809	Silvia	Yeah. Then work out how long the sides need to be.
810	Binny	Would that make it hold two cups?
811	Silvia	No. Rachael, where's the graph paper?

Learning, in this case could be classified as constructivist because it involved an unrestricted interplay between reality and possibility (Inhelder &Piaget, 1958).

Negotiation of meaning

Such incidents demonstrate the negotiation of meaning necessary for the completion of the task as well as the development of complex mathematical ideas. In reviewing transcripts of the groups at work in classrooms where children were free to take control of their own mathematical activity, it is easy to find examples of conjecture, reasoning, explaining, negotiation of both vocabulary and meaning, planning, generalising and responding. These established genres of social interaction are too often restricted by teachers taking strong control of the creation and supervision of tasks.

Each participant in the last discussion above contributed to its movement towards clarification of the task and the creation of possible solutions to the problems posed along the way. Individuals responded appropriately when others had not followed their reasoning and used alternative communication (sketches, hand movements, pointing to and handling the pyramid) to help others construe meanings which were shared enough to allow the discussion to continue. Choice of direction and ownership of the mathematics being explored rested firmly in the control of the learners. Such co-operation and freedom, however, were not enabled in the earlier examples because the teachers were perceived to be in control of instructional functions.

Students' notions of what it is to do mathematics in each of these classrooms would be likely to be quite different. Teacher H, while expecting students to make mathematical discoveries, planned the activity around these and his students accepted the implication that he had definite ideas about how tasks should be completed and the 'rules' which would evolve. Teacher N shaped the task carefully so that the activity would 'prove' the rule she planned to explain, and the students in her class provided no evidence of the notion that it is a student's role to discover mathematical ideas or to contribute to the learning of others. While there was knowledge development in these classrooms about the institutional and cultural contexts of schooling, there was little constitutive mathematical activity by students. In contrast, Teacher M's students looked for little direction and apparently thought it their task to negotiate and explore both the task and the resulting mathematical ideas. Mathematics, for them, was malleable: its meanings were to be construed.

CONCLUSION

The above snippets of classroom interaction are useful in examining the notion that knowledge originates from activity with objects. Many teachers have interpellated this claim by thinking that the provision of concrete materials or of hands-on activities is all that is required to relate mathematical concepts to the 'real world' of the child. However, we need to think of objects not as counters, blocks, cardboard, seed or other concrete materials. What students manipulate is developing mental constructs (Wheatley, 1991). Acting on the world may very well involve manipulating materials, but the objects of manipulation are also teachers' directions, written text, group discourses and perceptions of the roles of both teachers and learners in classrooms.

Also demonstrated above is how pedagogical moments can shape potential learning. Such moments, in combination, become processes of classroom interaction. Wheatley (1991) comments that with instructional processes which sanction natural instincts to construct meaning,

> Students come to realise they are capable of problem solving and do not have to wait for the teacher to show them the procedure or give them the official answer. Students come to believe that learning is a process of meaning-making rather than the sterile academic game of figuring out what the teacher wants. (p. 15)

The observations above, plus others recorded during the life of the project, support the claim of Candy (1989) that learners will not attain full and undisputed ownership over the learning situation while they construe the instructor as still exerting residual authority.

This claim has implications for our own practices as teacher educators. We criticize student teachers for not having clear learning objectives and well-sequenced lesson plans. We worry that teachers do not demonstrate learning sequences we believe are appropriate for different concepts. We model linear, transmissive curricula with carefully prepared tertiary programs. But it seems that presentation of loosely-defined tasks may allow for some exciting lessons about learning mathematics. This research suggests that we should lend more weight to Schratz' (in press) claim that

> The more interpretive space a teacher leaves in the learning process, the more likely the chances will be for the students to use their own knowledge of the world and everyday reasoning to tackle educational problems. However this process is also dependent on providing materials that can be subject to negotiation and multiple interpretations.

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